2021 James S. Rickards Fall Invitational

Algebra I Individual Solutions

- 1. A. Use the distance formula to calculate the radius of the dots: $\sqrt{((4 (-13))^2 + (18 29)^2)} = \sqrt{410}$. To find the area of a circle, apply the formula $\pi r^2 = (\sqrt{410})^2 \pi = 410\pi$.
- 2. C. $\frac{6x 18z = 66}{6} = x 3z = 11.$ $\frac{14x 8y 2z = -30}{2} = 7x 4y z = -15.$ $\frac{8x + 12y + 10z = -12}{2} = 4x + 6y + 5z.$ To use the elimination method, multiply the second equation by 3 and the third equation by 2 to cancel out the y terms. You are left with 29x + 7z = -57 and x 3z = 11. You can now solve the 2 variable system to get x = -1 and z = -4. Substitute those values into any of the prior equations to get y = 3. $\frac{xy}{z} = \frac{3}{4}$
- 3. E (1609). Use either long or synthetic division to obtain the remainder.
- 4. **B**. $343a^3 + 27b^6$ is a sum of cubes. $a^3 + b^3 = (a+b)(a^2 ab + b^2)$. In this case, you would find $((7a)^2 (7a)(3b^2) + (3b^2)^2)$ which equals $49a^2 21ab^2 + 9b^4$.
- 5. A. Make $a = \sqrt{2} + \sqrt{3}$. You would then have $\frac{7}{a \sqrt{5}}$. Multiply the numerator and denominator by the conjugate, $7(a + \sqrt{5})$

 $a + \sqrt{5}$, to get $\frac{7(a + \sqrt{5})}{a^2 - 25}$. Substitute the value of a back into the fraction to get $\frac{7(\sqrt{30} + 2\sqrt{3} + 3\sqrt{2})}{12}$.

- 6. C. There are 8 integers (-9, 2, 0, -1, -29, 3, 1, -8) and 3 natural numbers (1, 3, 2). 8 x 3 = 24.
- 7. **D**. Gabbi can eat $\frac{1}{14}$ of the bag in 1 min. Navya can eat $\frac{1}{21}$ of the bag in the same amount of time. Together, they can eat $\frac{1}{14} + \frac{1}{21} = \frac{5}{42}$ of the bag in 1 minute. $\frac{5}{42}(6) = \frac{5}{7}$ which is how much of the bag they can finish in 6 minutes. $1 \frac{5}{7} = \frac{2}{7}$ which is how much of the bag is left.
- 8. A. The number of ways to rearrange the letters without any restrictions $=\frac{8!}{2!}=20160$. The number of ways to rearrange the letters so that the E and K are always next to each other $=\frac{7!}{2!}=2520$. 20160 2520 = 17640 which is the number of ways where the E and K are apart.
- 9. **B**. The series is an infinite geometric series. The formula to find the sum of an infinite geometric series $=\frac{a}{1-r}$ where a is the 1st term and r is the common ratio. The 1st term = 2 and the common ratio $=\frac{2}{3}$. Plug the values into the formula to get 6 as the answer.
- 10. C. Firstly, isolate the y in Navya's line to get the equation y = 6x + 3. To find the intersection points, equate the 2 expressions to get $3x^2 15x + 21 = 6x + 3$. Bring everything to one side to form a quadratic equation. $3x^2 21x + 18 = 0$. The roots of the equation are 1 and 6. Substitute those values back into the equation to get 9 and 39, respectively, as the y coordinates, or ordinates, of the intersection points. $3y \times 9 = 351$.
- 11. E (-160). Put the equation into vertex form: $\frac{y = -18x^2 72x 32}{-18} > \frac{-y}{18} + 4 = x^2 4x + 4 + \frac{16}{9} = \frac{-y}{18} 4 = (x 2)^2 + \frac{16}{9} 4$. Multiply both sides by -18 to get $y = -18(x + 2)^2 + 40$. The vertex is (-2, 40). 2(-2)(40) = -160.
- 12. **D**. To find the inverse, swap the x and y in the function and then solve for y: $x = \frac{3y^3 10}{2}$. Multiply both sides by 2, add 10 and divide by 3 to get $y^3 = \frac{2x + 10}{3}$. Finally, take the cube root of both sides to get $y = \sqrt[3]{\frac{2x + 10}{3}}$ which is the inverse function. Plug 8.5 into the equation to get $\sqrt[3]{9}$.
- 13. **B**. $24^C x = 10,626 = \frac{24!}{(24-x)!(x!)} = 10626 >x = 4$. 4 is the total amount of people going to the concert, including Manjari. Subtract 1 to get the number of friends she is taking which is 3.
- 14. A. 40 people = dueling. 31 = sword-making. 55% of 40 is 22. 22 people participate in both dueling and sword making, meaning 31 22 = 9 people do ONLY sword-making.

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15. E $(-\infty, -4) \cup (2, \infty)$. To find the domain of this function, you must consider the possible restrictions, particularly of the denominator. We cannot have the denominator be 0 nor can we let it be negative as we are raising it to the power of $\frac{1}{2}$. Thus we have, $2x^2 + 4x - 16 \ge 0$. If you solve the inequality, you should get x < -4 and x > 2.

16. **D**.
$$f^2(g(3)) = f(f(g(3))) = f(f(\sqrt{43})) = f(32) = 1013.$$

- 17. C. The midpoints of each of the sides of ABCD is (5, 7), (6, 2), (2, 1), and (1, 6). Use the distance formula to get the perimeter of the shape formed by those points.
- 18. A. $\frac{1}{9^2} = 3^{2x-7} = \frac{1}{3^4} = 3^{2x-7}$ Cross multiply to get $1 = 3^{2x-3} 2x 3 = 0$. $x = \frac{3}{2}$.
- 19. C. Expand to get $5x^2 10x \ge 3x 12 + 6 = 5x^2 13x + 6 \ge 0$. If you solve this inequality, you get $x \le \frac{3}{5}$ and $x \ge 2$.
- 20. E (4). $(a+bi)^2 = 48-64i >a^2 + 2abi b^2 = 48-64i$. Equate the real parts to the real number and the imaginary part to the imaginary number: $a^2 b^2 = 48$ and 2abi = -64i. a = 8 and b = -4. 8 4 = 4.
- 21. **B**. $\frac{(sleep)(netflix)}{cheetos} > \frac{(7)(4)}{11} = \frac{10x}{22}$. Solve for x to get 5.6.
- 22. E $(2\sqrt{7})$. $\sqrt{8+2\sqrt{7}}+\sqrt{8-2\sqrt{7}}=x$. Square both sides to get $8+2\sqrt{7}+8-2\sqrt{7}+2\sqrt{(8+2\sqrt{7})(8-2\sqrt{7})}=x^2$. Simplify: $16+2\sqrt{64-28}=x^2=28$. Isolate x by taking the square root of both sides which gets you $2\sqrt{7}$.
- 23. **D**. The degree of $f(x) \cdot g(x) =$ the sum of the degrees of f(x) and g(x) = 2 + 3 = 5. The constant term = 15 $\times -9 = -135.5 135 = -130$.
- 24. C. Rationalize the denominators of each of the fractions. You will find that all the terms cancel out except the 7 and $-3\sqrt{5}$.
- 25. C. Nonagon = 9 sides. To find the interior angle measure, use the formula $\frac{180(n-2)}{n}$ where n is the number of sides. Plug in 9 to get 140. Divide 140 by 2 to get 70. The complement of 70 = 90 70 = 20. The supplement of 20 = 180 20 = 160.
- 26. **A**. Use the function that corresponds to the correct input value. $f(3) = 7, f(-6) = 187, f(f(-4)) = \frac{70}{3} \cdot 7 187 + \frac{70}{3} = -\frac{470}{3}$.
- 27. A. The maximum can be found by first finding $-\frac{b}{2a} = \frac{17}{5}$. Then plug $\frac{17}{5}$ into the quadratic to find the maximum height the ball can reach.
- 28. **B**. There are two cases: |4x 9| + 5 = -17 and |4x 9| + 5 = 17. Solve both cases to get $x = -\frac{13}{4}$ and $\frac{21}{4}$, respectively.
- 29. **D**. To find the slope of the perpendicular bisector, find the negative reciprocal of $\frac{-12-14}{5-(-9)}$. We have $y = \frac{7x}{13} + b$. We need b, aka the y- intercept of the line. Plug in the midpoint of the line segment, (-2, 1), to solve for b. You should get $\frac{27}{13}$. Our final answer is $y = \frac{7x}{13} + \frac{27}{13}$.
- 30. E (2112). The factors of 2021 are 1, 43, 47, and 2021. Add them up to get 2112.